5-2020

The Effects of Math Manipulatives In the Classroom

Emily Rowly

The Effects of Math Manipulatives In the Classroom

Emily Rowley

The University of Maine Farmington

Abstract

This study is a quantitative study of the correlation between student achievement and the use of individual math toolkits. Research indicates that the use of manipulatives has a positive effect on student learning in math classrooms. However, studies are limited to manipulative use in one unit of study or one grade level. In addition, researchers and teachers use only one type of manipulative when looking for growth in student achievement. In this study, students with individual toolkits comprised of many manipulatives from their program of study were followed over a nine-week period. Students had access to manipulatives and exhibit voice and choice when choosing manipulatives to help make meaningful math connections. Students were given a pretest in which percentages are compared and cross-referenced with a log of daily toolkit use. Teachers were also surveyed about toolkit promotion during the course of the nine-week study. This study was conducted to add to current research concerning manipulative use in the mathematics classroom.

Mathematics achievement in the United States has been under scrutiny for decades. And for decades, educators have grappled with how to make math concepts clear and comprehensible for students. One method educators use is math manipulatives. The use of manipulatives (or concrete models) in the math classroom has been explored and researched at length. Groups such as the National Council of Teachers of Mathematics (NCTM) have placed emphasis on using manipulatives by listing “Use and connect mathematical representations” as one of their eight effective teaching practices (NCTM, 2014). Common Core State Standards places “model with mathematics” among the list of student practices. Over the years, educators have subscribed to the idea that “representations play an important role in deepening student learning of mathematics” (NCTM, 2017). Yet, even with this cultural shift of manipulative use, the Program for International Student Assessment (PISA) ranked Americans 36th out of 79 countries in math literacy, performing below the international average (Barshay, 2019). With manipulatives researched, explored at length, and stocked on the shelves of teachers’ classrooms, students still struggle to use manipulatives to make genuine connections in mathematics.

Literature Review

What are Manipulatives?

Manipulatives or concrete models are defined as “a mathematical idea by means of three-dimensional objects” (Fenemma, 1972, p.17) or “objects that students can grasp with their hands” (Clements, 1999, p. 46). Manipulatives can be as simple as on-hand items such as paper clips or buttons, but “commercially manufactured products designed to meet general or specific educational aims are also widely available" (Holms, 2013, p. 1).

The Success of Manipulatives in the Classroom

Research and articles championing the use of math manipulatives in the classroom can be found as early as 1949, with Henry Van Engen stating, "Meaning of words cannot be thrown back on the meaning of other words. When the child has seen the action and performed the act for himself, he is ready for the symbol for the act" (1949, p. 347). There is evidence to support that when students connect concrete models to abstract mathematics, their understanding is deepened and they are able to move flexibly between the two (van Engen, 1949; Fennema, 1972; McClung, 1998). The ability to understand and use symbols as a shorthand in mathematics is not inherent; students must first understand what the symbol represents. "This means, that most children in the elementary school years often require the use of concrete materials to make symbolic materials meaningful" (Fennema, 1972, p. 18). When manipulatives are purposefully and thoughtfully chosen, authentic connections can be made through the action of touching and moving. In a meta-analysis written by Amy Holms, in which 14 studies and 1,126 students were analyzed, manipulative use compared to nonuse in classrooms showed an effect size of 0.22, an achievement gain of 9% (Holms, 2013). With medium effect sizes reported out by researchers (Hattie et al., 2017; Ojose & Sexton, 2009), adopting manipulatives into the classroom seems to be a foregone conclusion.

Teacher Training

Where the use of manipulatives is found to be successful, there is stated a correlation between teacher training and student use (Bryant, 1992; Puchner, Taylor, O'Donnell & Flick, 2008; Ball, 1992). In a study by Veronica Bryant, students experienced an academic improvement when using manipulatives and strategies that teachers were trained to use through an in-service workshop and monthly professional development. The study indicates that when teachers were trained with materials and students have access to them, grades on report cards and standardized scores improve (Bryant, 1992). Varie Hudson Hawkins, who wrote a dissertation on the effects of math manipulatives on student achievement and, conducted a quantitative study resulting in manipulative use having nonsignificant results on achievement. It is important to note, however, in this study, teachers were not specifically trained to use any manipulative. Results showed that the more experienced teachers in the experimental group had better success with student achievement when students used manipulatives (Hawkins, 2007). This indicates that teacher knowledge could be a factor in the success of students making connections between abstract and concrete mathematics. Douglas Clements supports this idea by adding that the benefit of manipulatives can be seen across "grade level, ability level, and topic, given that use of a manipulative 'makes sense' for that topic" (Clements, 1999, p. 45). Not only should teachers be trained to use the tool, but they should also be trained when to use the tool. "Specifically, teachers need support making decisions regarding manipulative use, including when and how to use manipulatives to help them and their students think about mathematical ideas more closely" (Puchner, Taylor, Odonnell, & Fick, 2008, p. 323). In studies where manipulative use showed negative results, possible factors included teachers' lack of knowledge on the tool and students' lack of familiarization with the tool (McClung, 1998; Hawkins, 2007).

How Manipulatives are Used

An emerging pattern in this literature shows teachers and researchers use math manipulatives in isolation, gauging the benefit of the manipulatives from an isolated unit of study or one school year. During the time of research, only one type of manipulative is offered. Teachers will pass out the manipulative at the beginning of a lesson and collect it at the end. Many studies cite manipulatives used in this way (Couture, 2012; Hawkins, 2007; Bryant, 1992). Isolation within teaching and using manipulatives doesn't allow the student to make connections between manipulatives and concepts, nor does it give an appreciation of how manipulatives can apply to all areas of mathematics, increasing flexibility in thinking. Once the referenced studies have been completed, students' continued use of the manipulative is unclear. Also unclear is the fate of the manipulative itself. There is a need for research in which the participants have access to meaningful manipulatives daily, in ways in which they choose manipulatives they feel will help them make sense of math problems. In short, there is a need for research that studies the effects of student ownership over math manipulatives and its correlation with student achievement.

Purpose

The purpose of this study was to investigate the correlation between student academic success and daily access to manipulatives in an individual “math toolkit.” This is significantly different from current research in that students will have ownership of manipulatives (individual toolkits v. community collection) and access to tools in daily instruction. In addition, many tools are available to the students, instead of one teacher-chosen tool. This study is designed to answer the question: Will student achievement improve with daily access to many different manipulatives? And, does student ownership of manipulatives increase use of the manipulatives? While the purpose of the study is very clear, the outbreak of COVID-19 shifted the focus of this study. Without the ability to give students a posttest, the purpose of the study shifted to analyzing how students answered questions on the pretest and how many times were students using their toolkits. During the timeframe in which data was collected, teachers were offered half-hour training sessions each week for the purpose of supporting teachers in the meaningful use of toolkits. For this study, “meaningful use” includes building the use of toolkits into lesson plans, prompting students to use toolkits, and creating purposeful opportunities for students to use their toolkits. The variable of teacher training raises a question of the impact of effective teacher training on manipulatives, which is a variable for future isolation and research.

Method

Design

This is a correlational study that seeks to investigate daily student toolkit use with academic achievement through a prediction design. I predict if students have ownership over manipulatives, they will be more likely to use the manipulatives regularly in the classroom (daily or multiple times a week).

Hypothesis

Students who use toolkits daily will show more growth in achievement than their peers who do not use toolkits daily.

Participants

Three teachers volunteered for this study; two first grade teachers and one second-grade teacher. Teachers were recruited by email and asked to consider participating in the study. The offer to participate was not extended to grades three through four due to the amount and type of materials on hand. Rumford Elementary School (RES) was the focus school due to accessibility to the teachers and students (I work in this building). RES is part of a larger district, RSU #10, consisting of three elementary schools, two middle schools, and two high schools. Informed consent was obtained from the appropriate parties.

Measures

It is important to note that toolkits have been implemented at RES for three years. Grade 2 participants have used toolkits in some form since their kindergarten year (2017-2018). Grade 1 participants have used toolkits since their kindergarten year (2018-2019). This study was the first attempt to measure the effectiveness of toolkits.

Once teachers and students were identified, students were given a pretest at the start of the research project (first-grade assessment, Appendix A, and second-grade assessment Appendix B). When the pretest was given, students were not allowed to use their toolkits. The original work of this study was to, after nine weeks of toolkit use, have students take a posttest using toolkits. The pre- and posttest would then have been compared to see what, if any, correlation existed between toolkit use and academic achievement. Since the posttest couldn't be given, the data was analyzed from a lens of how the students answered questions. Four categories were created to analyze story problems answered by students: Answer Correct, Answer Present in Solution, Thinking Shown, and Strategy Shown. Embedded in this study, though not the focus, is the idea that training teachers to use manipulatives has an effect on how often students will choose to use their toolkits. Teachers were offered a half-hour training session weekly. The session featured a tool that is common in all toolkits, kindergarten through second grade (ten-frames, number lines, rekenreks, subitizing cards, 2-dimensional shapes, 3-dimensional shapes, etc.) and instruction regarding how teachers can make meaningful connections between the manipulative and student learning.

Data Collection

Assessments (consisting of story problems) were coded with the following language: Correct Answer, Answer Present in Solution, Thinking Shown, and Strategy

Shown. Also, during the course of nine weeks, students were asked to track toolkit use by placing their names on a chart after each math lesson. This list of names was compiled daily, resulting in a master list of participants and their daily usage of toolkits. The last two pieces of data collection were the number of times a teacher attended a manipulative training session and teacher surveys. The teacher survey was created in Google Forms and disseminated via email. Both pieces (training attendance and surveys) was collected weekly over a nine-week period

Data Analysis

Story problems were coded by what the student showed when answering. *Correct Answer* indicates the answer was correct, either explicitly written or communicated as the result of an equation. *Answer Present in Solution* indicates the reader could find the answer in the solution, either as a part of the picture or a part of the equation. If the answer was a part of the equation, but not the result, the answer was considered not correct. Figures 1 and 2 show how answers can be present in the solution but still leave the student with an incorrect answer.

Figure 1

Second Grade Item 3b Part Part Whole with Part Unknown Story Problem

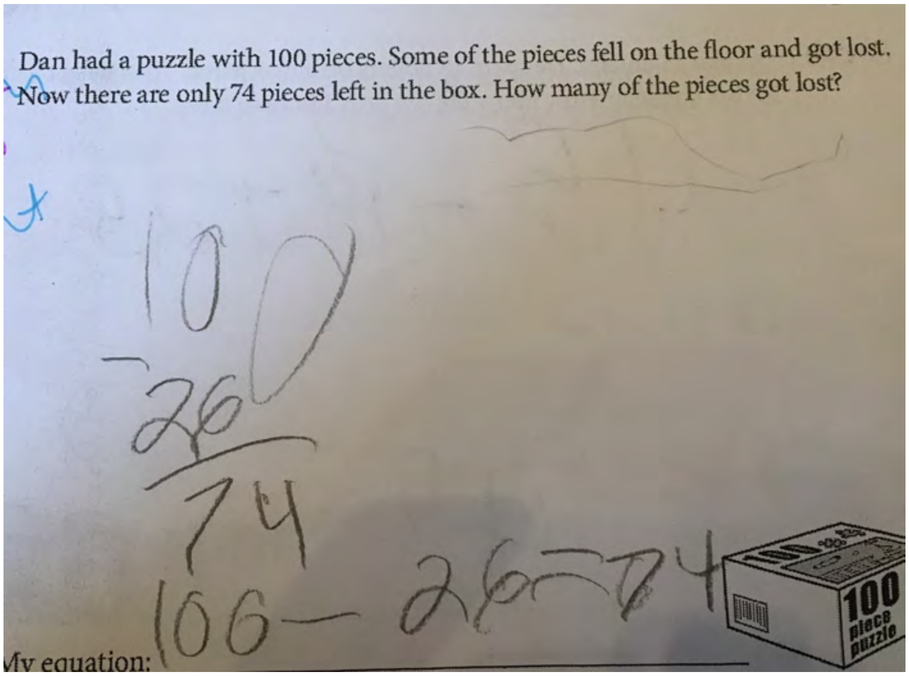
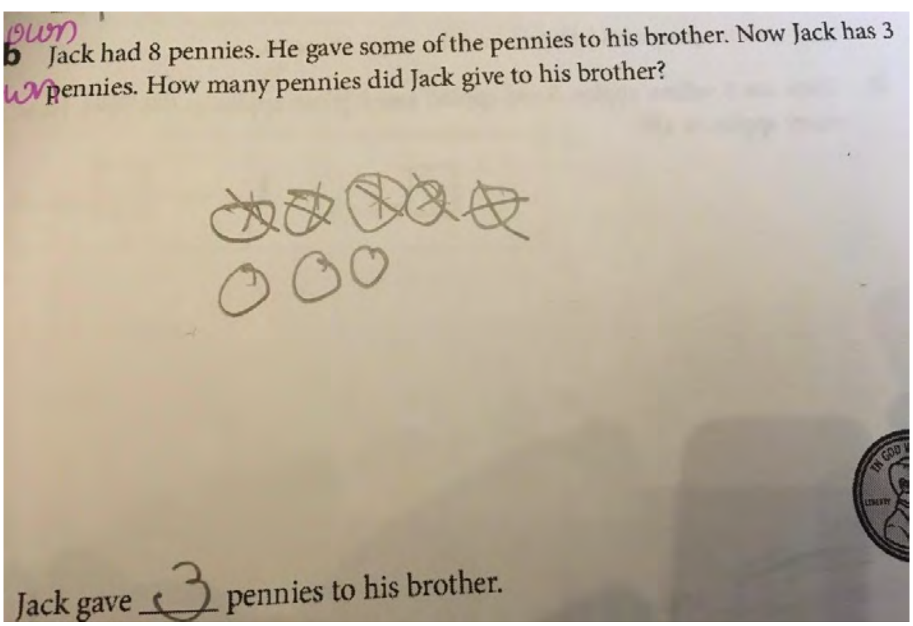


Figure 2

First Grade Item 2b Separating with Change Unknown Story Problem



Thinking Shown indicates that students communicated whether they considered the story problem as a story about joining or a story about takeaway or comparison. This could be communicated through an equation with the addition or subtraction symbols or pictures in which they would add more items to the first group drawn or cross off/erase from the group that was drawn. S*trategy shown* indicates that the reader can see how the student solved the problem. In Figure 2, the student crossed out all but three pennies. This student's strategy was to look at the whole amount of pennies and separate the whole into parts; one part of pennies that Jack has and one part that Jack gave away.

First-grade students completed six story problems:

1. One joining story with the change unknown (item 1a)
2. One joining story with the result unknown (item 1b)
3. One separating story with the result unknown (item 2a)
4. One separating story with the change unknown (item 2b)
5. One comparison story with the difference unknown (item 3a)
6. One comparison story with the quantity unknown (item 3b)

Second-grade students completed 8 story problems:

1. One joining story with the start unknown (item 1a)
2. One separating story with the start unknown (item 1b)
3. Two comparison stories with the quantity unknown (items 2a and 2b)
4. Two part/part/whole stories with the parts unknown (items 3a and 3b)
5. One joining story with the result unknown (item 4a)

6. One multi-step story with the result unknown (item 4b)

All posttests were to be coded using the following language: Correct Answer, Answer Present in Solution, Thinking Shown, and Strategy Shown. A growth score would have been calculated by comparing each coded category of the pretest score to that of the posttest score. A student would show growth in a category by scoring a higher percentage on the posttest than on the pretest. A growth score would have been cross-referenced with a student's percentage of daily toolkit use. The percentage of daily toolkit use is calculated by the number of days a student used the toolkit divided by the number of days of the study. If there is a positive correlation between student toolkit usage and student achievement (growth in the coded categories), then one could conclude that individual toolkit use positively affected student achievement.

Results

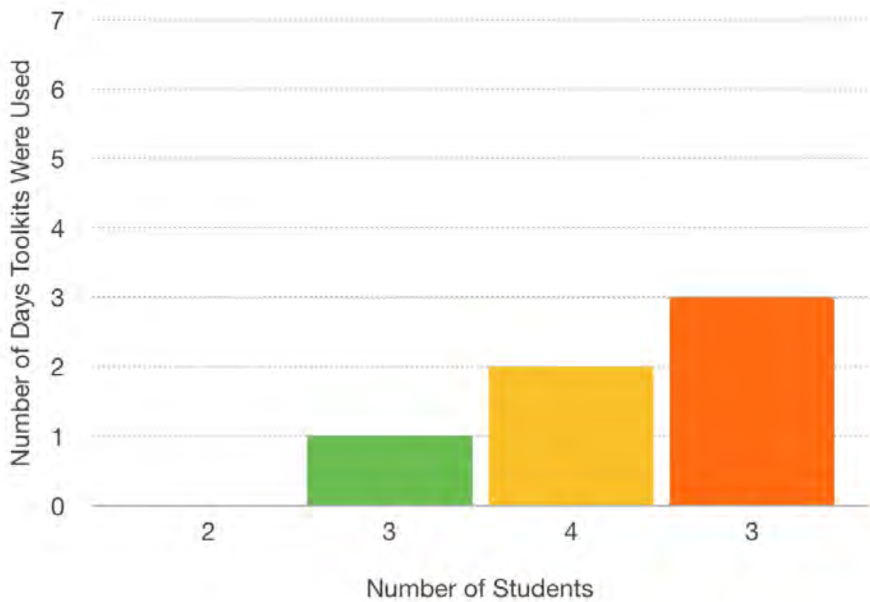
Due to the outbreak of COVID-19, this research project could not be concluded in the same manner as the outset. Included with this paper, however, is data that was collected until March 13, 2020, one week prior to the closing of our school.

Group 1

Group 1 is a first-grade classroom consisting of 12 students; 25% female, and 75% male. Girls make up the population of students who used toolkits most frequently. When given the opportunity, males used their individual toolkits no more than two days, with two males not using their toolkits once in seven days. Both males and females used toolkits less than half the opportunities that were provided to them. Table 1 shows the number of times students in Group 1 used their toolkits given seven opportunities.

Table 1

Days of Toolkit Use for Group 1



As shown in Table 2, Group 1 was most successful in answering items 2a and 2b with a 92% success rate. In addition, items 2a and 2b showed the highest rate of students showing their thinking and/or the strategy used to solve the problem.

Table 2

Response Components of Pretest Story Problems Items 2a & 2b

Group 1	Item 2a	Answer Correct	Answer Present in Solution	Thinking Shown	Strategy Shown	Item 2b	Answer Correct	Answer Present in Solution	Thinking Shown	Strategy Shown
Student 1				x	x			x	x	x
Student 2		x		x			x		x	x
Student 3		x					x			
Student 4		x					x			
Student 5		x					x			
Student 6		x					x			
Student 7		x					x			
Student 8		x					x		x	
Student 9		x					x			
Student 10		x		x			x			
Student 11		x					x			
Student 12		x		x	x		x		x	x
Total		11	0	4	2		11	1	4	3
Percentage		92%	0%	33%	17%		92%	8%	33%	25%

Through informal conversations, the teacher from Group 1 disclosed appreciation of the tools, but a dislike of the individual toolkit, expressing that each student having their own box of tools is hard to manage logistically. This teacher declined two opportunities to attend a toolkit training session. Tables 2 and 3 show the results of the pretest. Group 1 had the lowest toolkit usage, but the highest percentage of correct answers on the pretest. In addition, Group 1 had the lowest percentage of students showing their thinking and/or strategy use.

Table 3

Response Components of Pretest Story Problems Items 1a, 1b, 3a and 3b

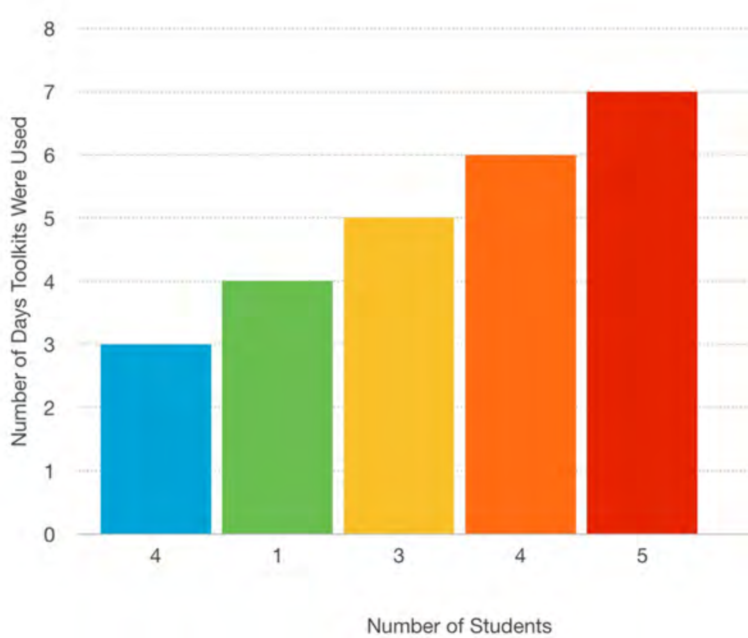
Group 1	Item 1a	Answer Correct	Answer Present in Solution	Thinking Shown	Strategy Shown	Item 1b	Answer Correct	Answer Present in Solution	Thinking Shown	Strategy Shown	Item 3a	Answer Correct	Answer Present in Solution	Thinking Shown	Strategy Shown	Item 3b	Answer Correct	Answer Present in solution	Thinking Shown	Strategy Shown
Student 1		x															x			
Student 2		x					x					x					x			
Student 3		x					x					x					x			
Student 4		x					x					x					x			
Student 5		x					x					x					x			
Student 6																	x			
Student 7		x		x			x					x								
Student 8		x					x					x								
Student 9												x								
Student 10		x		x			x													
Student 11												x					x			
Student 12							x					x					x		x	
Total		8	0	2	0		8	0	0	0		9	0	0	0		8	0	1	0
Percentage		67%	0%	17%	0%		67%	0%	0%	0%		75%	0%	0%	0%		67%	0%	8%	0%

Group 2

Group 2 is a first-grade classroom consisting of 17 students; 59% female and 41% male. Males make up the group of students who used their individual toolkits half or less of the offered opportunities. Table 4 shows the number of times students in Group 2 used their toolkits given eight opportunities.

Table 4

Days of Toolkit Use for Group 2



Performance of Group 2 on the pretest shows that (with the exception of one instance on item 1a), females were the only group to show their thinking and/or strategy. While males did answer story problems correctly, only one male showed evidence of thinking and strategy on one item. As a group, 88% answered item 2b correct, their highest percentage on an item. The teacher from Group 2 is a female who has been teaching for 6 years. Through informal conversations, the teacher from Group 2 disclosed appreciation of individual toolkits for students. This teacher states that students engaged with their toolkits independently and especially enjoyed the discourse when offered opportunities to talk about which tool they used to solve a problem. The teacher from Group 2 declined two opportunities to attend a toolkit training session.

Group 3

Group 3 is a second-grade classroom consisting of 11 students; 45% female and 55% male. Toolkit use by gender is evenly distributed, although the student using their toolkit only once within the three days was male. Tables 5 and 6 show that all students who answered an item communicated how they perceived the story problems. While students wrote equations that were true, many students wrote their equations incorrectly, with the result of the equation not being the answer to the story problem. Students consistently demonstrated their thinking through the use of equations which were a requirement of the directions. Only two students left answers blank, both were female.

Table 5

Response Components of Pretest Story Problems Items 1a, 1b, 2a and 2b

Group 3	Item 1a	Answer Correct	Answer Present in solution	Thinking Shown	Strategy Shown	Item 1b	Answer Correct	Answer Present in solution	Thinking Shown	Strategy Shown	Item 2a	Answer Correct	Answer Present in solution	Thinking Shown	Strategy Shown	Item 2b	Answer Correct	Answer Present in solution	Thinking Shown	Strategy Shown
Student 1			x	x			x		x			x		x	x		x		x	
Student 2			x	x	x			x	x	x		x		x	x		x		x	x
Student 3		x		x	x		x		x					x				x	x	
Student 4			x	x	x				x			-	-	-	-		-	-	-	-
Student 5			x	x	x			x	x	x		x		x	x		x		x	x
Student 6			x	x	x				x			x		x	x				x	
Student 7			x	x	x			x	x	x			x	x	x				x	x
Student 8		x		x	x		x		x	x		x		x			x		x	
Student 9			x	x	x			x	x	x		x		x	x		x		x	x
Student 10			x	x				x	x			x		x			x		x	
Student 11			x	x	x		x		x	x		x		x			x		x	
Total		2	9	11	9		4	5	11	6		8	1	10	6		7	1	10	4
Percentage		18%	82%	100%	82%		36%	45%	100%	55%		73%	9%	91%	55%		64%	9%	91%	36%

Table 6

Response Components of Pretest Story Problems Items 3a, 3b, 4a and 4b

Group 3	Item 3a	Answer Correct	Answer Present in solution	Thinking Shown	Strategy Shown	Item 3b	Answer Correct	Answer Present in solution	Thinking Shown	Strategy Shown	Item 4a	Answer Correct	Answer Present in solution	Thinking Shown	Strategy Shown	Item 4b	Answer Correct	Answer Present in solution	Thinking Shown	Strategy Shown
Student 1			x	x				x	x					x					x	
Student 2				x	x				x	x				x	x				x	x
Student 3				x					x					x					x	
Student 4		-	-	-	-		-	-	-	-		-	-	-	-	-	-	-	-	-
Student 5			x	x	x				x	x				x	x				x	x
Student 6				x					x			x		x					x	
Student 7				x	x				x	x		-	-	-	-		-	-	-	-
Student 8				x					x					x	x				x	x
Student 9		x		x	x		x		x	x			x	x	x				x	x
Student 10				x				x	x			x		x					x	
Student 11			x	x	x		x		x	x		x		x					x	
Total		1	3	10	5		2	2	10	5		3	1	9	4		0	0	9	4
Percentage		9%	27%	91%	45%		18%	18%	91%	45%		27%	9%	82%	36%		0%	0%	82%	36%

Through informal conversations, the teacher from Group 3 disclosed appreciation of individual toolkits and admitted to struggling to structure math class in a way that gives students opportunities to use them. This teacher also admitted that the use of toolkits feels like an extra step in an already fast-paced and cramped curriculum. Lastly, the teacher and I had candid conversations about how the tools engaged lower-performing students, providing them an avenue to answer questions and a forum for mathematical discourse as they share answers and explain how tools are used. The teacher from Group 3 declined one opportunity to attend a toolkit training session.

Discussion

This study set out to answer questions about individual toolkit use in public education classrooms. Often in classrooms, teachers keep community tools. These tools belong to the classroom and to the teacher and are used year after year with new students. Individual toolkits differ in that they go and grow with an individual student.

Students collect and use tools throughout the academic year. These tools and manipulatives are pieces that complement the curriculum. At the end of the academic year, students physically bring their toolkits to the following grade. This study was set to answer questions on whether ownership of mathematics manipulatives and tools improves engagement with manipulatives and whether they improve academics. Due to the outbreak of COVID-19, these research questions could not be answered. With schools being shut down, data collection stopped. However, data collected may be examined and conclusions drawn regarding individual and community tool use.

Limitations

A few limitations should be considered when drawing conclusions about the data gathered for this study. Teachers did not start gathering data at the same time. Specifically, the teacher from Group 1 had more days to offer opportunities for toolkit use than the teacher of Group 2 or Group 3. Story problems for both the first and second grades were conducive to number plucking, in which students could hone in on keywords and pluck the numbers to perform an operation. Attendance wasn't tracked. Some students didn't use toolkits because they were absent from school, not because they were denied an opportunity for use. Group 3 has had individual toolkits for the longest period of time - three academic years. Groups 1 and 2 have used individual toolkits in some form for two years.

Implications

Considerations for future teaching and learning include taking a closer look at how toolkits impact students' ability to communicate mathematical thinking and their use of a strategy to solve a problem. Data shows that Group 1 took advantage of using their toolkits the least, less than half the offered times. Also, Group 1 did not show thinking or strategies to solve problems on the pretest. Group 3 had ten students use their toolkits more than half of the opportunities offered. On all items, some students showed their strategy for solving, and all students who answered a story problem communicated how they thought about the problem through an equation. Is there a correlation between toolkit use and the ability to communicate mathematical thinking? If so, the use of individual toolkits could enhance teachers' ability to tease out misconceptions and give meaningful feedback. Another consideration for teaching and learning is whether gender plays a role in toolkit use. Data from this study indicate that males are less likely to use toolkits than females. This is implied in all three groups, with males representing the group that took advantage of opportunities to use toolkits less frequently. In Group 2, none of the males indicated how they thought about a particular problem or demonstrated their strategy to solve the problem; they give a single answer. Are males less likely to show their thinking and not use tools because they can visualize the problem mentally? Or is there a gender bias with the toolkits, either within the creation of the tool/manipulative or demonstrated unknowingly by teachers?

Another piece for teachers to consider is the structure of story problems that students are solving. First graders were very successful with the story problem structured as a separating problem with the change unknown. Is this structure the easiest for them or was it the content of the story problem that made sense to them? In other words, does experience with the content (in this case pennies) matter in order to make sense of what is happening in the story?

Lastly, why does Group 3 have correct parts and wholes in their equations, but not the correct result, and does it matter? Through a wider lens, it shouldn't matter that Group 3 didn't write equations that matched the story problems. It should matter that they made sense of the problem in a meaningful way.

Research on this topic should be continued in order to reveal a clearer picture of the use of individual toolkits over the use of community tools. All three groups have had toolkits for multiple years. Not all teachers implement toolkits to the same degree, a condition that has been observed, but not studied until now. The teacher from Group 1 has historically struggled with buy-in for individual toolkits. During the course of the year, students were offered limited opportunities to use toolkits, a pattern that continued as the study commenced. Even without meaningful use of toolkits, Group 1 scored higher on every item than Group 2. In addition to this data, Group 3, who has had toolkits the longest, consistently showed their thinking when solving story problems. A minimum of four students showed their strategy on every problem and some students showed their answers in their solution, implying that they had an understanding of how to solve the story problem, but lacked an understanding of how to show their answer as the result of an equation. As the data stands, there is no evidence to support that toolkit use advanced academic knowledge; to the contrary, as the group with the highest level of correct answers was also the group that used their toolkits the least. However, this doesn't speak to the level of growth students may experience over time. Nor does it speak to how teachers can change their instruction to diversify teaching and learning as they reflect on how students think about math and understand mathematics.

Conclusion

While the results of this study couldn't be concluded, the need for further research is apparent. It remains undetermined whether individual toolkits are more academically valuable than community tools or if students feel more ownership over the individual tools than they would community tools. This research may be useful as a prompt for further study, a reason to remain cognisant of how gender may play a role in toolkit use, or a reason to reflect upon the role of procedure in a world where thinking and product are becoming a focus.

References

Bryant, V. A. (1992). *Improving mathematics achievement of at-risk and targeted students in grades 4-6 through the use of manipulatives.* Retrieved from ERIC database. (62836792; ED355107).

Clements, D. H. (2000). ‘Concrete’ Manipulatives, Concrete Ideas. *Contemporary Issues in Early Childhood*, *1*(1), 45–60. doi: 10.2304/ciec.2000.1.1.7

Couture, K. (2012). *Math manipulatives to increase 4th-grade student achievement*. Retrieved from ERIC database.

Engen, H. V. (1949). An Analysis of Meaning in Arithmetic. II. *The Elementary School Journal*, *49*(7), 395–400. doi: 10.1086/459064

Ball, D. L. (1992). Magical hopes: Manipulatives and the reform of math education. *American Educator: The Professional Journal of the American Federation of Teachers, 16*(2), 14-18. Retrieved from ERIC database.

Barshay, J. (2019, December 17). What 2018 PISA international rankings tell us about U.S. schools. Retrieved December 18, 2019, from https://hechingerreport.org/what-2018-pisa-international-rankings-tell-us-about-u-s-schools/.

Creswell, J. W., & Guetterman, T. C. (2019). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research*. New York, NY: Pearson.

Fennema, E. H. *(1972).* Models and mathematics*. The Arithmetic Teacher, 19(8), 635-640.*

Hattie, J. A. C., Fisher, D., Frey, N., Gojak, L. M., Moore, S. D., & Mellman, W. (2017). *Visible learning for mathematics: grades K-12: What works best to optimize student learning*. Thousand Oaks (California): Corwin Mathematics.

Hawkins, V. H.. (2007) *The effects of math manipulatives on student achievement in mathematics.* Retrieved from ERIC database.

Holms, A.B. (2013). Effects of manipulative use on PK-12 mathematics achievement: A meta-analysis. *Society for Research on Educational Effectiveness,* 1-18.

McClung, L. W. (1998). *A study on the use of manipulatives and their effect on student achievement in a high school algebra I class.* Retrieved from ERIC. (62468568; ED425077).

NCTM, National Council of Teachers of Mathematics. (2014). *Principles to actions: ensuring mathematical success for all*. Reston, VA.

NCTM, National Council of Teachers of Mathematics. (2017). *Taking action: Implementing effective mathematics teaching practices.* Reston, VA.

Ojose, B., & Sexton, L. (2009). The effect of manipulative materials on mathematics achievement of first grade students. *The Mathematics Educator*, *12*(1), 3–14. doi: 10.13189/ujer.2015.030206

Puchner, L., Taylor, A., Odonnell, B., & Fick, K. (2008). Teacher learning and mathematics manipulatives: A collective case study about teacher use of manipulatives in elementary and middle school mathematics lessons. *School*

Science and Mathematics, *108*(7), 313–325. doi: 10.1111/j.1949-8594.2008.tb17844.x

Standards for Mathematical Practice. (n.d.). Retrieved from http://www.corestandards.org/Math/Practice/.

The Math Learning Center. (2019). *Bridges in Mathematics: Comprehensive Growth Assessment.* Retrieved from https://bridges.mathlearningcenter.org/curriculum

Uribe-Flórez, L. J., & Wilkins, J. L. M. (2010). Elementary school teachers' manipulative use. *School Science & Mathematics*, *110*(7), 363–371. https://doi-org.ursus-proxy-6.ursus.maine.edu/10.1111/j.1949-8594.2010.00046.

Appendix A

1 Solve each story problem below. Use numbers, pictures, or words to help solve the problem and show your thinking. Write your answer on the line.

a Six penguins were swimming in the water. Some more penguins jumped in. Then there were 10 penguins. How many more penguins jumped into the water to swim?

_____ more penguins jumped into the water to swim.

b There are 3 yellow apples, 6 red apples, and 7 green apples in the bowl. How many apples in all?

There are _____ apples in all.

2 Solve each story problem below. Use numbers, pictures, or words to help solve the problem and show your thinking. Write your answer on the line.

a Eleven butterflies were in the garden. Eight of the butterflies flew away. How many butterflies were left in the garden?

_____ butterflies were left in the garden.

b Jack had 8 pennies. He gave some of the pennies to his brother. Now Jack has 3 pennies. How many pennies did Jack give to his brother?

Jack gave _____ pennies to his brother.

3 Solve each story problem below. Use numbers, pictures, or words to help solve the problem and show your thinking. Write your answer on the line.

a Jon has 6 berries. Della has 10 berries. How many more berries does Della have than Jon?

Della has _____ more berries than Jon.

b James and Kim are picking flowers. Kim has 5 more flowers than James. James has 3 flowers. How many flowers does Kim have?

Kim has _____ flowers.

Appendix B

1 Solve each story problem below. Use numbers, pictures, or words to show your thinking. Write an equation to match the problem.

a Jeff had some baseball cards. His friend gave Jeff 8 more baseball cards. Then Jeff had 14 baseball cards. How many baseball cards did Jeff have to start?

My equation: ______________________________

b There were some butterflies in the garden. Six of the butterflies flew away. Then there were 7 butterflies left in the garden. How many butterflies were in the garden before?

My equation: ______________________________

2 Solve each story problem below. Use numbers, pictures, or words to show your thinking. Write an equation to match the problem.

a Kim and Sara are picking up shells. Kim has 9 more shells than Sara. Sara has 5 shells. How many shells does Kim have?

My equation: ______________________________

b Pablo and Carlos are picking flowers for their mom. Pablo has 10 fewer flowers than Carlos. Carlos has 18 flowers. How many flowers does Pablo have?

My equation: ______________________________

3 Write an equation to match each story problem. Solve the equation. Use numbers, pictures, or words to show your thinking.

a Jade and Luis have been saving pennies. They are going to put their pennies together into one jar. Jade has some pennies. Luis has 36 pennies. When they put their pennies together, they will have 81 in all. How many pennies does Jade have?

My equation: ______________________________

b Dan had a puzzle with 100 pieces. Some of the pieces fell on the floor and got lost. Now there are only 74 pieces left in the box. How many of the pieces got lost?

My equation: ______________________________

Appendix C

Weekly Teacher Survey

Your email address (**erowley@rsu10.org**) will be recorded when you submit this form. Not you?
Switch account

* Required

Date Today *

Your answer

This week, how many math lesson plans included a toolkit activity? *

Choose

This week, how many days did one or more student use a toolkit without being prompted by the teacher? *

Choose

This week, how many times was the class given purposeful opportunities to use their toolkits *

Choose

SUBMIT